Finding The Charge To Mass Ratio of an Electron

Experimental Physics Report

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1 Introduction

This experiment was based on J.J. Thomson's 1897 experiment.[1] In his experiment, Thomson shot a beam of particles through a partial vacuum in a glass tube and between two plates. He noticed that the particles, or electrons as we now know them, were being deflected at an angle away from the negatively charged plate allowing him to conclude that the particle had a negative charge. Using magnets, he was also able to calculate the charge to mass ratio for these particles. The goal of this experiment was to find a ratio of electron charge to electron mass. The basic physics principal that was used in this experiment was that charged moving particles get deflected in magnetic fields and thus when the strength of the magnetic field and the distance of the deflections is known or measured, then the charge to mass ratio can be calculated. A uniform magnetic field can be achieved with the Helmholtz coils.

2 Theory

Starting with Lorentz's force law[2] given in Equation 1,

$$\vec{F}_m = q\vec{v} \times \vec{B} \tag{1}$$

where $\vec{F}_m$ is the magnetic force vector, q is the charge of the particle, $\vec{v}$ is the velocity vector of the particle, and $\vec{B}$ is the magnetic field vector, the equation can be simplified to Equation 2, which is scalar, if the assumption is made that the the electron beam and magnetic field are perpendicular.

$$F_m = qvB \tag{2}$$

Assuming that the path of a charged particle, an electron in this experiment, is perfectly circular in a uniform B-field, the centripetal force must also be considered and factored in. Centripetal force is given by Equation 3

$$F_c = \frac{mv^2}{r} \tag{3}$$

where m is the mass of the particle, v is the velocity of the particle, and r is the radius of the circular path it travels on.

Since the electron path was constant in this experiment, it is assumed that $F_{net} = 0 = F_m - F_c$ and as a result $F_c = F_m$. All other forces, like gravity, can

be assumed to be negligible. Therefore combination of Equation 1 and 2 results in equation 4.

$$\frac{mv^2}{r} = qvB \tag{4}$$

Equation 4 can be solved for the charge to mass ratio producing Equation 5.

$$\frac{v}{rB} = \frac{q}{m} \tag{5}$$

Equation 5 shows that only 3 variables are needed to calculate e/m ratio. The r can be measured from visible results of the experiment while the v and B are a little bit more involved to calculate. Since the electrons are being accelerated by some electrical potential, V, their kinetic energy is equal to q*V resulting in the Work-Energy theorem shown in Equation 6 which can then be rearranged to solve for velocity, v, as shown in Equation 7.

$$qV = K.E. = \frac{1}{2}mv^2 \tag{6}$$

$$v = \sqrt{\frac{2qV}{m}} \tag{7}$$

The B-field produced by the Helmholtz coils is given by Equation 8, where I is the current through the coils, N is the number of turns in each coil equal to 130, μ_0 is the permeability constant equal to $4\pi * 10^{-7} H/m$, and A is the radius of the Helmholtz coils, which was measured to be 0.154 ± 0.005 m.

$$B = \frac{[N\mu_0]I}{(5/4)^{3/2}A} \tag{8}$$

Deriving the B-field in the Helmholtz coils was a much more involved process.[2] Starting with Biot-Savart law, shown in Equation 9, where r is the distance from the source to some point, dl is some small length along the loop, we can derive Equation 10 which is Biot-Savart law for a loop by substituting $r^2 = x^2 + A^2$, since the field is going along the x axis and $\int dl = 2\pi r$, since the path is one loop.

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \times \vec{r}}{r^2} \tag{9}$$

$$B(x) = \frac{\mu_0 I}{2} \frac{A^2}{(A^2 + x^2)^{3/2}} \tag{10}$$

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Therefor, the magnetic field along the x axis of loops located at -d and +d is Equation 11, which equals 0 at x=0.

$$B_x = \frac{\mu_0 I}{2} \left(\frac{A^2}{(A^2 + (x+d)^2)^{3/2}} + \frac{A^2}{(A^2 + (x-d)^2)^{3/2}} \right) \tag{11}$$

Differentiating the expression produces Equation 12:

$$\frac{dB_x}{dx} = \frac{\mu_0 I}{2} \left(-\frac{3}{2}[A^2 + (x+d)^2]^{-5/2} 2(x+d) - \frac{3}{2}[A^2 + (x-d)^2]^{-5/2} 2(x-d) \right) \tag{12}$$

Equation 12 can then be simplified to produce Equation 13:

$$\frac{dB_x}{dx} = -\frac{3\mu_0 I A^2}{2} \left(\frac{x+d}{[A^2 + (x+d)^2]^{5/2}} + \frac{x-d}{[A^2 + (x-d)^2]^{5/2}} \right) \tag{13}$$

Differentiating $\frac{dB_x}{dx}$ again results in $\frac{d^2 B_x}{dx^2} = 0$ as demonstrated in Equation 14:

$$\frac{d^2 B_x}{dx^2} \bigg|_{x=0} = 0 = \left[\frac{(A^2 + (x+d)^2)^{5/2} - \frac{5}{2}(x+d)(A^2 + (x+d)^2)^{3/2}(x+d)2}{(A^2 + (x-d)^2)^5} \right]$$
$$+ \left[\frac{(A^2 + (x-d)^2)^{5/2} - \frac{5}{2}(x-d)(A^2 + (x-d)^2)^{3/2}(x-d)2}{(A^2 + (x-d)^2)^5} \right] \tag{14}$$

Simplifying Equation 14 yields Equation 15:

$$0 = \left[\frac{(A^2 + d^2)^{5/2} - 5d^2(A^2 + d^2)^{3/2}}{(A^2 + d^2)^5} + \frac{(A^2 + d^2)^{5/2} - 5d^2(A^2 + d^2)^{3/2}}{(A^2 + d^2)^5} \right] \tag{15}$$

Multiplying Equation 15 by $(A^2 + d^2)^5$ and dividing by $(A^2 + d^2)^{3/2}$ produces Equation 16 which can then be solved do obtain the value of d, $d = \frac{A}{2}$.

$$0 = A^2 + d^2 - 5d^2 \tag{16}$$

Plugging in that value of d into Equation 11, produces Equation 17 which is equal to the magnetic field of a Helmholtz coil with only one wire turn in the coil. Multiplying Equation 17 by N produces Equation 8, which accounts for N turns in each coil.

$$B_x \bigg|_{x=0} = \frac{\mu_0 I A^2}{2} \left(\frac{1}{(A^2 + (\frac{A}{2})^2)^{3/2}} + \frac{1}{(A^2 + (\frac{A}{2})^2)^{3/2}} \right) = \frac{\mu_0 I}{(5/4)^{3/2}A} \tag{17}$$

Now since both B and v are known, plugging in Equation 8 and Equation 7 into Equation 5 yields the final formula for calculating charge to mass ratio as shown in Equation 18.

$$\frac{q}{m} = \frac{v}{Br} = \frac{2V(5/4)^3 A^2}{(N\mu_0 Ir)^2} \tag{18}$$

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3   Methods / Experimental Setup

Setup and the collection of data was fairly simple in this experiment. Even though the experiment was conducted in a fairly dim room, the hood was placed over the apparatus, as shown in Figure 1, to help with visibility. The E/M apparatus was
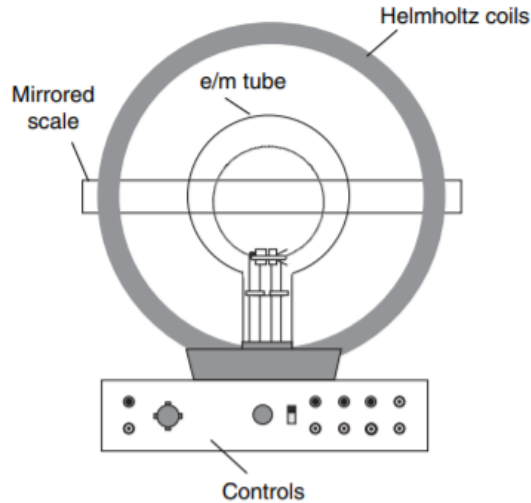

Figure 1: E/M Apparatus

connected to a power source, shown in Figure 2, and the toggle switch was switched to the e/m MEASURE option and all meters and connections were connected as shown while the current adjust knob for the Helmholtz coils was off. The Heater was set to $6.30 \pm 0.005V$ on the power supply while the Electrodes received varying voltages from 150 V to 300 V, shown in Table 1 and Table 2, as this was the independent variable of the experiment. The Helmholtz coils were adjusted to $6.17\pm 0.005V$ which produced a current of $1.394 \pm 0.0005A$ according to a digital multimeter. Next, power to the power supply was turned on. The apparatus was then allowed to heat up for 10 minutes. The final step was to adjust the bulb, shown in Figure 3 so that the green ring inside from the electrons would be parallel to the coils.

3.1   Data Collection

This experiment was conducted as one set of 11 measurements at different voltages and the collected data was put into Table 1.
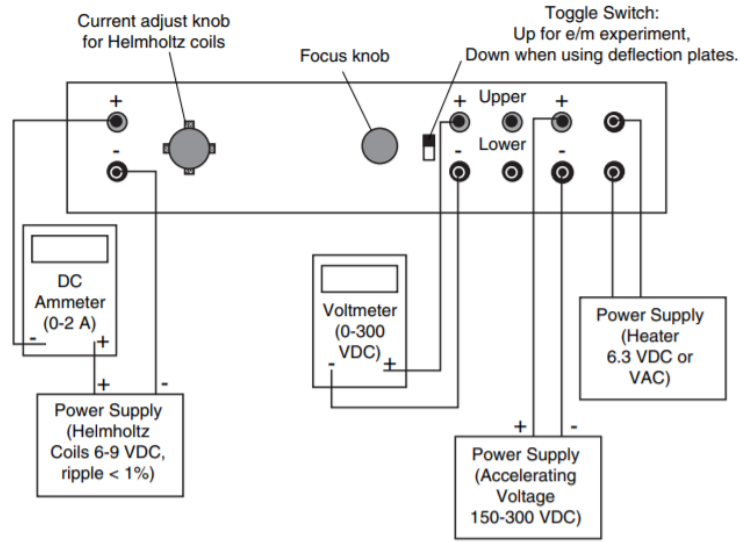
Figure 2: Power Source and Connections

Current to Heater (A)	Voltage to Electrodes (V)	Radius of Electron Ring (m)
1.394 $\pm$0.0005	155.3 $\pm$0.05	3.30 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	165.4 $\pm$0.05	4.35 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	180.1 $\pm$0.05	4.40 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	195.4 $\pm$0.05	4.75 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	210.7 $\pm$0.05	4.95 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	225.6 $\pm$0.05	5.20 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	240.4 $\pm$0.05	5.40 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	255.8 $\pm$0.05	5.55 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	270.4 $\pm$0.05	5.75 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	285.2 $\pm$0.05	5.80 $\pm0.05 \times 10^{-2}$
1.394 $\pm$0.0005	300.0 $\pm$0.05	5.90 $\pm0.05 \times 10^{-2}$

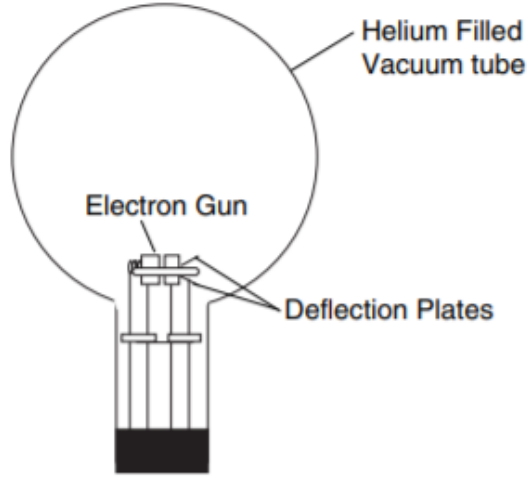Table 1: Measured radius based on constant current and changing voltage

Figure 3: E/M Tube

4 Results

Using the data provided in Table 1, the following charge to mass ratios were calculated and put into Table 2. However, we noticed that the very first value produced a much larger result for E/M. This caused us to calculate all values twice, once with all 11 values and once with only 10 values, omitting the measurement at 155.3 V, to see if uncertainty and results can be improved. Based on the results in Table 2, the $1.5 \pm 0.3 \times 10^{11}$ C/kg value for 11 values and the $1.4 \pm 0.05 \times 10^{11}$ C/kg value for 10 values are consistent with one another and are a little bit lower than the generally accepted value of $1.75882001076(53) \times 10^{11}$ C/kg. However, using the standard deviation as the uncertainty gives a extremely wide range of values since it is 19% and 4% error for the calculation with all 11 values and calculation with only 10 values.

Using a different approach to error analysis yielded much better uncertainty. Substituting "X" for q/m in Equation 18 and separating out the radius results in Equation 19.

$$X = \frac{2V(5/4)^3 A^2}{(N\mu_0 I)^2 r^2} \tag{19}$$

Solving for r^2 yields Equation 20.

$$r^2 = \left[\frac{2(5/4)^3 A^2}{(N\mu_0 I)^2 X} \right] * V \tag{20}$$

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Voltage (V)	E/M (C/kg)	E/M (C/kg)
155.3	$2.4 \pm 0.04 \times 10^{11}$	—
165.4	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
180.1	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
195.4	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
210.7	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
225.6	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
240.4	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
255.8	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
270.4	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
285.2	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
300.0	$1.4 \pm 0.04 \times 10^{11}$	$1.4 \pm 0.02 \times 10^{11}$
Average	1.5×10^{11}	1.4×10^{11}
Standard Deviation	3×10^{10}	5×10^{9}
% Uncertainty	19%	4%
Slope From Graph	1.16×10^{-5}	1.18×10^{-5}
Uncertainty in Slope	3.2×10^{-7}	1.2×10^{-7}
% Uncertainty in M	3%	1%
% Uncertainty in A	0.3%	0.3%
% Uncertainty in I	0.04%	0.04%
% Uncertainty in X (E/M)	3%	1%
Uncertainty in X (E/M)	4×10^{9}	2×10^{9}
X (E/M)	$1.428 \pm 0.004 \times 10^{11}$	$1.405 \pm 0.002 \times 10^{11}$

Table 2: Calculated E/M values and calculated uncertainties

Assuming the charge to mass ratio (X) and current (I) are constant, M can be substituted for $\frac{2(5/4)^3 A^2}{(N\mu_0 I)^2 X}$ producing a linear equation that relates r^2 to V where $r^2 = M * V$. Graphing both versions of the data in Excel produces graphs in Figures 4 and 5 shown with a proportional trend line since the radius is 0 at 0 volts.

Using Excel's LINEST function, the error in slope was found to be 3.2×10^{-7} for all data and 1.2×10^{-7} for the filtered data or 3% and 1% respectively. The LINEST function uses many underlining assumptions to produce the given error in slope. First, the function assumes that the relationship between r^2 and V is linear
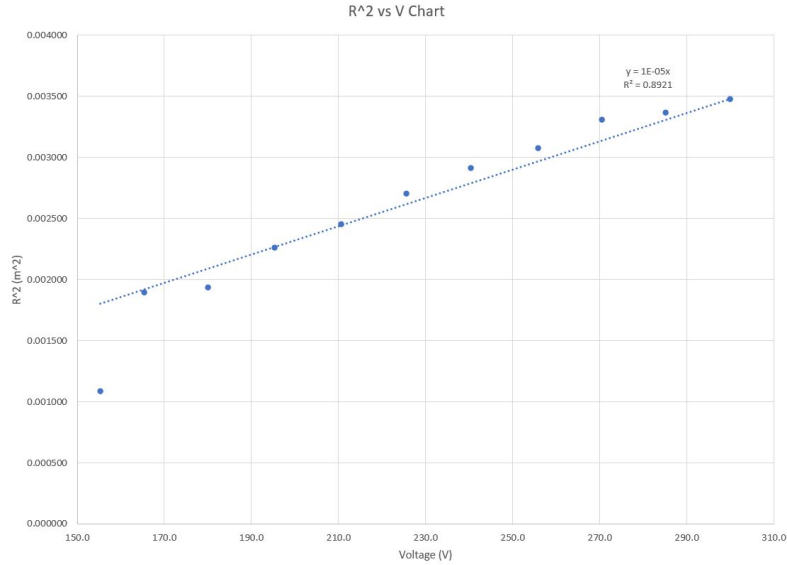
Figure 4: V vs r^2 graph for all data

and goes through the point, (0,0). This means that our equation can be simplified to the form of y=Bx or $r^2 = MV$ in our specific case as explained above. If x and y have no uncertainties to them, then the slope will be a straight line with perfect fit. Since there are uncertainties in y and x in real world measurements, the data when graphed, will be slightly off of the line of best fit. To get this line of best fit, it is assumed that while x has uncertainty, it is negligible. This is a reasonable assumption since usually the uncertainty of one variable dominates the other. Another assumption is that the measured dependent variable is spread with a Gaussian distribution, and thus the closer values to the true value appear more often then bad values. Applying theses assumptions produces the uncertainty in y as:

$$\sigma_y = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}(y_i - Bx_i)^2}$$

The N-2 comes from the fact that when there are only two points, uncertainty can not be determined. There are always two less degrees of freedom then there are number of data values. Knowing σ_y allows for the calculation of uncertainty in slope, B.

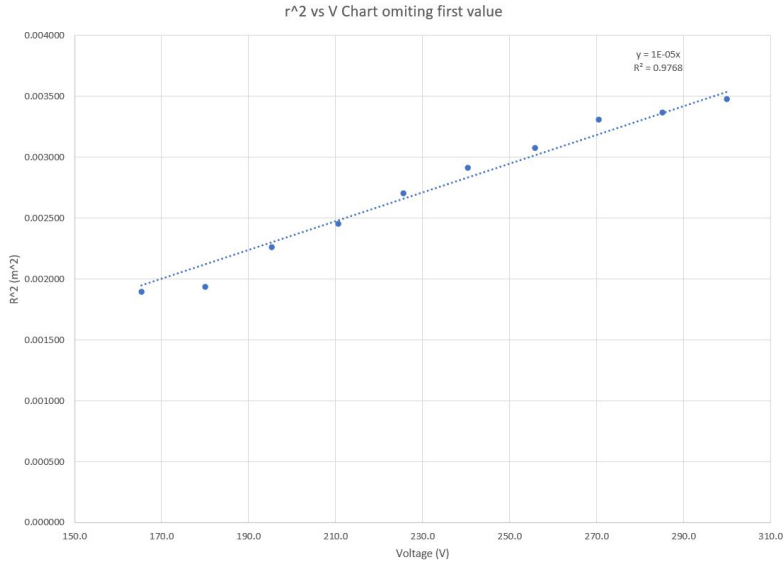$$\sigma_B = \sigma_y\sqrt{\frac{N}{N\sum x^2 - (\sum x)^2}}$$

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Figure 5: V vs r^2 graph for data without the first measurement

. This is how Excel's LINEST function gets those values for slope and the uncertainty in slope.[3]

Using the quadratic sum[3], errors in radius of the Helmholtz coils (A), and error in current (I), Equation 21 is produced.

$$\frac{\delta X}{X} = \sqrt{2\left(\frac{\delta I}{I}\right)^2 + 2\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta M}{M}\right)^2} \tag{21}$$

Using $5 \times 10^{-4} A$ as the uncertainty in current (I), 0.154 ± 0.005 m as the value and uncertainty of the Helmholtz coil radius (A), and the the previously stated values for uncertainty in slope (M), Equation 21 gives errors of 3% for all data and 1% for the selected data. Solving the substitute equation for slope, M, from Equation 20 for X produces Equation 22.

$$X = \frac{2(5/4)^3 A^2}{M(N\mu I)^2} \tag{22}$$

Plugging in the slopes from the graphs in Figures 4 and 5 along with all of the other known variables, $X = 1.428 \pm 0.004 \times 10^{11}$ C/kg for all the data and $X = 1.405 \pm 0.002 \times 10^{11}$ C/kg for the selected data. These values along with the uncertainty were also included in Table 2.

5 Conclusion

The results obtained from this experiment were a little bit less than the generally accepted value for the charge to mass ratio. Using the average and standard deviation method for calculating the ratio, the result was within the uncertainty but it is also worth noting that the uncertainty was 19% and 4% which is not all that great. Using the second method were the voltage was graphed with radius squared of the electron beam circle, the results were a little bit lower, at $1.428 \pm 0.004 \times 10^{11}$ C/kg for all the data and $1.405 \pm 0.002 \times 10^{11}$ C/kg for the selected data, rather than the generally accepted value of $1.75882001076(53) \times 10^{11}$ C/kg.

There are a good number of errors that could have contributed to the lower calculated value. First, the scale for reading off the radius for the charge circle was placed well behind the bulb. This definitely introduced paralax error that was random. Another contributing factor to a lower charge to mass ratio was that the charges were emitting visible light which means that energy was being lost by the charges in order for them to be observed. This light emission was measured to be 3.01 μW which would certainly add to the lower than expected charge over mass. This would be consistent with our results in that we did get a lower value than expected. The tube was also filled with helium[1]. This means that the charges were not passing through a vacuum and therefore they surely interacted and collided with the helium inside the bulb.This energy loss is modeled in Figure 6.
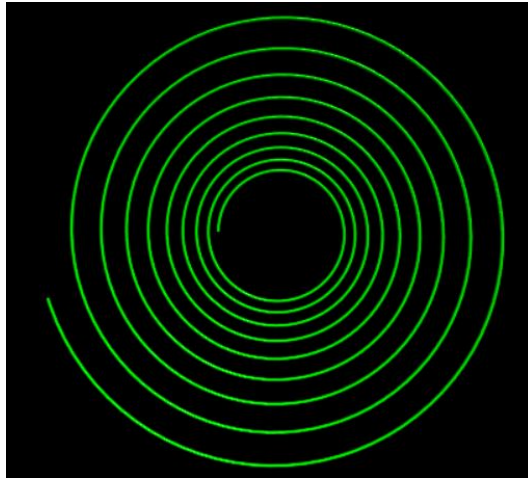

Figure 6: Simulation of Electron losing velocity and thus being less and less affected by B-Field

The last error that could probably be neglected was that while the Helmholtz coils were producing a very strong and uniform magnetic field, the other electronic devices in the room were also producing magnetic fields and some of those may have influenced the experiment in some form. The earths magnetic field was negated by lining up the Helmholtz coils perpendicular to the earths north to south poles. this means that the cross product between the electrons and the earth's B-field is 0. Further experimentation in this sphere of physics could include replacing the Helmholtz coils with elliptical coils and seeing how that would effect both the path of the charges and also how the equations would change to accommodate the new non-symmetry in the coils.

References

[1] PASCO, *Experiment Guide for the PASCO scientific Model SE-9638: E/M Apparatus* (PASCO).

[2] G. J. David, *Introduction to Electrodynamics*, 4th ed. (Cambridge University Press, 2017).

[3] J. R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed. (University Science Books, 1996).

A   Appendix: Simulation Computer Code

Here is the code provided to simulate an electron losing energy and thus velocity due to collisions with He atoms and emission of light. Generic numbers were used in this simulations and the loss was not made to scale due to the severe complexity of the calculations for energy loss due to collisions with He atoms.

```
GlowScript 2.9 VPython
Bfield = vec(0, 0,.01)
q = 45
mass = 2
charge = sphere(pos = vec(-10,0,0), radius =0.3, color =color.
    green, make_trail = True)
tmax =250
t = 0
dt = 0.02
velocity = vec(0,3,0)
force=q*vec(velocity.y*Bfield.z-velocity.z*Bfield.y,-velocity.x*
    Bfield.z+velocity.z*Bfield.x,velocity.x*Bfield.y-velocity.y*
    Bfield.x)+.01*velocity
momentum=velocity*mass
while (t<tmax):
    rate(1000)
    force=q*vec(velocity.y*Bfield.z-velocity.z*Bfield.y,-
        velocity.x*Bfield.z+velocity.z*Bfield.x,velocity.x*
        Bfield.y-velocity.y*Bfield.x)+.01*velocity
    momentum=momentum+force*dt
    velocity=momentum/mass
    charge.pos=charge.pos+(momentum/mass)*dt
    t=t+dt
```